

Quarkonium production and NRQCD matrix elements

Adam K. Leibovich
Department of Physics, Carnegie Mellon University,
Pittsburgh, Pennsylvania 15213, USA

Most recent calculations of quarkonium production are based on the NRQCD factorization formalism. This formalism is reviewed. To make predictions about specific cross section, universal NRQCD matrix elements need to be extracted from experiments. Extractions from different experimental situations are compared, with some emphasis on the extraction from LEP.

1. INTRODUCTION

Quarkonium is an interesting system since the mass of the heavy quark is much larger than the QCD scale,

$$m_Q \gg \Lambda_{\text{QCD}}. \quad (1)$$

One would hope, therefore, that it would be possible to calculate reliably in perturbation theory. If that were the end of the story, things would be simple. The only difficulty, which makes quarkonium very interesting, is that there is another natural small scale in the problem, the relative velocity of the heavy quarks, v . For charmonium $v_c^2 \sim 1/4$, while for bottomonium $v_b^2 \sim 1/10$.

Since we have a multi-scale problem, with the hierarchy

$$m_Q \gg m_Q v \gg m_Q v^2 \sim \Lambda_{\text{QCD}}, \quad (2)$$

it is useful to use an effective field theory. The effective field theory appropriate for a system with two heavy quarks is Nonrelativistic QCD (NRQCD) [1,2]. As the name implies, there is a relativistic expansion in v , as well as the usual perturbative expansion in α_s .

Before the advent of NRQCD, quarkonium production and decay was calculated using the so-called "Color-Singlet Model" (CSM). In this model, the $\bar{Q}Q$ pair is treated in a color-singlet state, in the $v \rightarrow 0$ limit. It was discovered, however, that the CSM did not describe quarkonium production very well in certain circumstances. For example, the CSM prediction for ψ' production at the Tevatron is off by about a factor of

30! As we will see, NRQCD can easily explain the discrepancy in this situation [3].

Whether NRQCD is the correct effective field theory to be used for quarkonium is an interesting and important question. There are some potential problems comparing NRQCD predictions to data, in particular the polarization. NRQCD make a definite prediction that at large transverse momenta, ψ' should be nearly 100% polarized [4]. However, it appears that the data is not following that trend [5]. So it may be that the α_s and v expansions do not converge well for charmonium. This is not to say that NRQCD is incorrect. NRQCD is a valid effective field theory, in that as the mass m_Q goes to infinity, the theory correctly reproduces full QCD. It may be that v_c is too large, or $m_Q v_c^2$ too small, for NRQCD to be useful in describing charmonium production. One could still hope that it will work for bottomonium.

How is it possible to test NRQCD? There is the polarization discussed above [6]. Another way is to compare NRQCD matrix elements extracted from different experiments.

2. FACTORIZATION FORMALISM

The physical picture of production in NRQCD begins with a hard scattering, in which a $\bar{Q}Q$ pair are produced with any spin, angular and color quantum numbers. This process can be calculated in perturbation theory. Then the $\bar{Q}Q$ evolve into the final state quarkonium H in some non-perturbative fashion. This is encoded in

the NRQCD matrix elements, which must be extracted from experiment.

The NRQCD factorization formula is the mathematical realization of the above picture. A general production process for $i + j \rightarrow H + X$ can be written as

$$d\sigma = \sum_n d\hat{\sigma}(ij \rightarrow Q\bar{Q}[n] + X) \langle O^H(n) \rangle, \quad (3)$$

where $d\hat{\sigma}(ij \rightarrow Q\bar{Q}[n] + X)$ is the cross section for producing $Q\bar{Q}$ in state n by scattering i and j , calculable in a perturbative expansion in α_s and perhaps convoluted with parton distribution functions (PDFs). The long distance matrix elements encode the hadronization of the heavy quarks in state n into the final quarkonium state H . The matrix elements can be written as

$$\begin{aligned} \langle O^H(n) \rangle &= \langle 0 | \psi^\dagger \Gamma^n \chi | \sum_X H + X \rangle \\ &\quad \times \langle \sum_X H + X | \chi^\dagger \Gamma'^n \psi | 0 \rangle \end{aligned} \quad (4)$$

The Γ^n contains Dirac matrices, color matrices and derivatives. The NRQCD matrix elements scale with a definite power of v , determined by Γ^n , allowing the truncation of the v expansion. Again, we expect the velocity expansion will work better in the bottom sector than the charm sector. The factorization described above still needs to be put on firmer ground. We should keep in mind that when we use the NRQCD factorization formalism to predict production rates, we are not only testing the α_s and v expansions, but also the factorization formalism.

The scaling is determined by looking at what perturbations are necessary to give a non-vanishing result for the time ordered product. To give non-zero overlap, multipole moment interactions may need to be inserted. For example, the matrix element $\langle O_8^{J/\psi}(^3S_1) \rangle$ scales as v^4 compared to the $\langle O_1^{J/\psi}(^3S_1) \rangle$, since we need two $E1$ insertions in the amplitude. The first insertion changes L and neutralizes the color. The second changes L back to an S wave. Each insertion cost a factor of v in the amplitude, or a total of v^4 in the rate. The scalings of the most important matrix elements for ψ production are

$$\langle O_1^\psi(^3S_1) \rangle \sim v^0, \quad (5)$$

$$\langle O_8^\psi(^3S_1) \rangle \sim v^4, \quad (6)$$

$$\langle O_8^\psi(^3P_J) \rangle \sim v^4, \quad (7)$$

$$\langle O_8^\psi(^1S_0) \rangle \sim v^4. \quad (8)$$

We can relate the color-singlet matrix elements to the wave function at the origin, or derivatives of the wave function at the origin, using the vacuum saturation approximation

$$\langle O^H(n) \rangle \approx \langle 0 | \psi^\dagger \Gamma^n \chi | H \rangle \langle H | \chi^\dagger \Gamma'^n \psi | 0 \rangle, \quad (9)$$

dropping the sum over X . This allows us to use the lattice, data, or models to obtain the color-singlet matrix elements. It also means that the color-singlet model has been incorporated into the NRQCD factorization formalism.

A quick aside. Many people say the NRQCD model, or the “color-octet” model. However, NRQCD is not a model. The CSM, on the other hand, is a model, since there is no limit in which it will reproduce full QCD.

3. QUARKONIUM AT THE TEVATRON

As previously mentioned, the CSM fails to describe the data at the Tevatron. We will now discuss how NRQCD improves the situation. We will concentrate on J/ψ production. A similar analysis can be done for Υ production. See Refs. [7] for recent analyses.

At large transverse momentum, the leading order NRQCD prediction in α_s and v is $O(\alpha_s^3 v^0)$. This is the same as the leading order CSM prediction, which grossly underestimates the rate. Even the shape of spectrum is wrong, with the theory dropping as $1/p_\perp^8$ compared to the data which falls as $1/p_\perp^4$.

We can go to higher order in the perturbative expansion: $O(\alpha_s^5 v^0)$. In this case, fragmentation processes occur, which, due to some propagators being close to on-shell, can have large contributions. These fragmentation diagrams dominate over the lower order graphs at large p_\perp , with the theory now falling as $1/p_\perp^4$, similar to the data. However, the normalization of the theory is still far below the experiment.

This is as far as the CSM can go. To improve the prediction, NRQCD is needed [3]. Instead of just including higher order in α_s , we can also

Table 1

Table for $M_k^{J/\psi}$ in units of 10^{-2} GeV^3 extracted from the Tevatron data. The second set of errors, when present, are due to scale variation. The first set are statistical. k varies between 3 and 3.5.

| Ref. | MRSD0 | MRS(R2) | CTEQ2L | CTEQ4L | GRVLO | GRVHO |
|------|-----------------|----------------------------------|-----------------|---------------------------------|---------------------------------|-----------------|
| [8] | 6.6 ± 1.5 | - | - | - | - | - |
| [9] | - | $10.09 \pm 2.07^{+2.79}_{-1.26}$ | - | $4.38 \pm 1.15^{+1.05}_{-0.74}$ | $3.90 \pm 1.14^{+1.46}_{-1.07}$ | - |
| [10] | 1.32 ± 0.21 | - | 1.44 ± 0.21 | - | - | 0.60 ± 0.21 |
| [11] | 1.32 ± 0.21 | - | 1.32 ± 0.21 | - | - | 0.45 ± 0.09 |
| [12] | - | - | - | 6.52 ± 0.67 | - | - |

include higher order in v : $O(\alpha_s^3 v^4)$. Now there are color-octet fragmentation processes proportional to the matrix element $\langle O_8^\psi(^3S_1) \rangle$, which also scales as $1/p_\perp^4$. Of course, the value of the matrix element is unknown, so the fact that we can get a good fit really means that the extracted matrix element is not abnormally large or small.

The fragmentation contribution can be approximated as

$$d\sigma(p\bar{p} \rightarrow H) = \sum_i \int dz d\sigma_i D_{i \rightarrow H}, \quad (10)$$

where $d\sigma_i$ is the cross section to produce an on-shell parton i , $D_{i \rightarrow H}$ is the fragmentation function for parton i to produce quarkonium state H with momentum fraction z , and the sum is over partons. The advantage of writing it in this way is that it is easy to sum large logs of $\log p_\perp/m_Q$ using Altarelli-Parisi (AP) evolution.

At lower p_\perp , fragmentation no longer dominates, and it is necessary to include all non-fragmentation diagrams as well. Contributions due to other octet matrix elements, $\langle O_8^\psi(^1S_0) \rangle$ and $\langle O_8^\psi(^3P_J) \rangle$ are now important. These channels both fall as $1/p_\perp^6$, making it impossible to determine them independently. Instead, a linear combination of the two

$$M_k^H = \langle O_8^\psi(^1S_0) \rangle + \frac{k}{m_Q^2} \langle O_8^\psi(^3P_0) \rangle \quad (11)$$

is usually extracted.

At small p_\perp , higher twist contributions become important. These contributions scale as powers of $\Lambda/\sqrt{p_\perp^2 + m_Q^2} v^n$, where the value of n is not known. Also at small p_\perp , the rates tend to diverge, due to soft gluon effects. To handle these divergences, the low p_\perp region can just be ignored

or modeled by including initial parton showering/intrinsic k_\perp .

In Table 1, different extractions of $M_k^{J/\psi}$ are collected. The second set of errors, when present, are due to scale variation. The other errors are statistical. As can be seen, there is large uncertainty due to the choice of PDF. At this point, the best we can hope to say is that this combination of matrix elements is on the order of

$$M_3^{J/\psi} \sim \text{few } 10^{-2} \text{ GeV}^3. \quad (12)$$

Table 2 contains different extractions of $\langle O_8^{J/\psi}(^3S_1) \rangle$. Again the second set of errors, when present, are due to scale variation, while the other errors are statistical. Here the largest error is not due to the PDFs, but due to the scale variation. This is an indication that higher order perturbative corrections may be very large. Again we only know the matrix element is on the order of

$$\langle O_8^{J/\psi}(^3S_1) \rangle \sim \text{few } 10^{-3} \text{ GeV}^3. \quad (13)$$

4. LEP EXTRACTION

We would like to do better than the order of magnitude extractions discussed above at the Tevatron. Since the dominant errors are due to scale or PDF variation, it seems difficult make improvements at the Tevatron. We would also like to test the formalism, by comparing the matrix elements extracted from different experiments.

The natural place to look for cleaner extractions would be at an e^+e^- machine, since there is no PDF nor initial state gluon radiation to worry about. The prediction at CLEO does not have a large dependence on the color-octet matrix el-

Table 2

Table for $\langle O_8^{J/\psi}(^3S_1) \rangle$ in units of 10^{-3} GeV^3 extracted from the Tevatron data. The second set of errors, when present, are due to scale variation. The first set are statistical.

| Ref. | MRSD0 | MRS(R2) | CTEQ2L | CTEQ4L | GRVLO | GRVHO |
|------|---------------|-------------------------------|---------------|-------------------------------|------------------------------|---------------|
| [8] | 6.6 ± 2.1 | - | - | - | - | - |
| [9] | - | $14.0 \pm 2.2^{+13.5}_{-7.9}$ | - | $10.6 \pm 1.4^{+10.5}_{-5.9}$ | $11.2 \pm 1.4^{+9.9}_{-5.6}$ | - |
| [10] | 2.1 ± 0.5 | - | 3.3 ± 0.5 | - | - | 3.4 ± 0.4 |
| [11] | 6.8 ± 1.6 | - | 9.6 ± 1.5 | - | - | 9.2 ± 1.1 |
| [12] | - | - | - | 3.94 ± 0.63 | - | - |

ements away from the endpoint. Therefore LEP is the natural choice.

Formally there are two leading order contributions in the α_s and v expansion, both in the singlet channel, of order $O(\alpha_s^2 v^0)$. The contribution from gluon radiation in the singlet channel $Z \rightarrow \psi gg$ is suppressed by powers of M_ψ^2/E_ψ^2 [13]. The color-singlet charm quark fragmentation process $Z \rightarrow \psi c \bar{c}$ [14], which has no power suppression, dominates over non-fragmentation processes for large E_ψ . Light quark octet fragmentation (in which the mother parton does not combine to form part of the bound state) is naively of order $\alpha_s^2 v^4$, down by $v^4 \sim 1/10$ compared to charm fragmentation. However, this channel is enhanced due to the presence of large logs and a numerical factor of five due to the number of possible quarks that initiate the process [15]. The same logs that enhance the octet channel also put the convergence of the perturbative expansion into question.

The tree-level calculation of the differential cross section in the color octet production channel [15] scales as $z \rightarrow 1$ as

$$\frac{d\Gamma}{dz} \sim \alpha_s^2 \frac{\log(M_Z^2/M_\psi^2)}{z} \langle O_8^{J/\psi}(^3S_1) \rangle, \quad (14)$$

leading to large double logs in the total rate. Since $\alpha_s \log(M_Z^2/M_\psi^2) \approx 1.5$, we should treat $\alpha_s \log(M_Z^2/M_\psi^2)$ as order one and resum all powers of the large logarithm. With this counting, the octet channel is $O(\alpha_s^0 v^4)$, on par with the singlet fragmentation contribution. More practically, the tree-level calculation has a factor of two uncertainty associated with the scale at which α_s is evaluated, since $\alpha_s(M_\psi)/\alpha_s(M_Z) \approx 2$ (this is just a restatement that there is a large logarithm). The resummation of the leading logarithms re-

duces this uncertainty, so the resummation procedure is essential from both a practical and a formal standpoint.

To resum the logs, we write the rate in the fragmentation limit as

$$\begin{aligned} \frac{d\Gamma}{dz} = & \int_z^1 \frac{dy}{y} [2C_q(\mu^2, y) D_q(\mu^2, z/y) \\ & + C_g(\mu^2, y) D_g(\mu^2, z/y)] . \end{aligned} \quad (15)$$

We can now use AP to sum the logs of M_Z^2/M_ψ^2 .

However, summing the above mentioned logs will only yield the correct leading order differential rate if z is sufficiently large.¹ When z is parametrically small, terms of the form $\alpha_s \log(z)/z$ become just as important. Furthermore, these logs will also contribute double logs to the total rate given that the lower limit on z is $2M_\psi/M_Z$. This second type of log, due to soft gluon emission, is resummed using a formalism familiar from discussions of jet multiplicities [16], where the color-coherence of the soft gluon emission is very important.

It is possible to do both of these resummations [17], see Fig. 1. The rate depends on the linear combination

$$\begin{aligned} & \sum_m \langle O_8^{H(m)}(^3S_1) \rangle \times \text{Br}[H(m) \rightarrow J/\psi X] \\ & = (1.9 \pm 0.5_{\text{stat}} \pm 1.0_{\text{theory}}) \times 10^{-2} \text{ GeV}^3 \end{aligned} \quad (16)$$

since the data includes feeddown from higher charmonium states. It turns out that this leads to

¹At very large $z \sim 1$ we do not get the correct result since we have neglected the shape function which enters near the endpoint. This shape function has been argued to be the solution of the so called ‘‘Hera anomaly’’. It is not important here since the rate is very small in this region.

an extraction with much smaller theoretical uncertainty than the analogous value from the Tevatron $1.4 \pm 0.2_{stat} \pm 1.4_{theory}$. For the Tevatron value, the extraction from Ref. [8] was used for the central value.

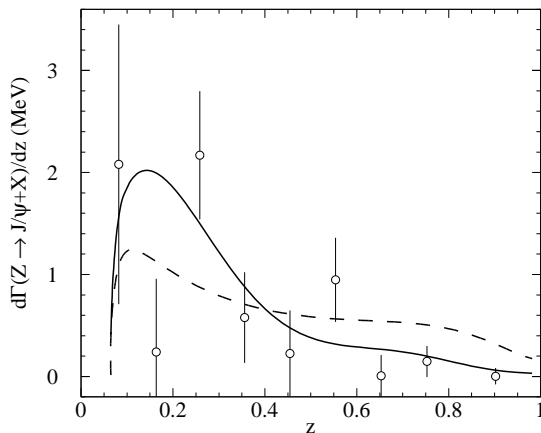


Figure 1. Differential rate $d\Gamma/dz$ as a function of $z = 2E_\psi/M_Z$ vs data. The dashed line is the sum of the tree-level octet and singlet results and the solid line is the octet plus the singlet resummation. From Ref. [17]

It is difficult to tell from Fig. 1 if there is much improvement in the prediction from this resummation. To make the comparison more quantitative, it is useful to take the ratio of the first moment to the zeroth moment

$$\frac{1}{\Gamma(Z \rightarrow J/\psi X)} \int dz z \frac{d\Gamma}{dz}(Z \rightarrow J/\psi X) \quad (17)$$

which for the resummed rate is 0.30. The tree level rate gives ~ 0.5 . A very rough estimate of this quantity obtained from the data [18] suggests a value of 0.26 ± 0.10 . This is in sharp contrast to the color singlet prediction. The tree-level color

singlet decay rate predicts the ratio of the first moment over the zeroth moment to be 0.62. Resummation softens the color singlet decay rate, but the ratio is still too large, 0.47. The ratio is independent of the color singlet matrix element. A rigorous extraction of the first moment by the experimental groups could provide an extremely clean, quantitative test of the NRQCD approach.

5. OTHER EXTRACTIONS

5.1. HERA

The NRQCD prediction does not seem to fit the data at HERA very well. As $z = E_{J/\psi}/E_\gamma$ goes to the endpoint, $z \rightarrow 1$, the NRQCD prediction begins to rise. This has been dubbed the “HERA anomaly”, and has been used to argue that NRQCD is incorrect.

However, it is now known that the NRQCD prediction breaks down near the boundary of phase space, precisely where there is a problem at HERA [19,20]. In this region the velocity expansion is breaking down, and a shape function must be introduced. In fact, one should not compare the prediction with the data above $z \sim 1 - v^2 \approx 0.7$. If one looks only below this point, there is no problem with the theory. Unfortunately, it is not really possible to extract the matrix elements from the data in this region.

5.2. $B \rightarrow J/\psi$ DECAYS

For $B \rightarrow J/\psi$ decays, the first worry is that the corrections to factorization will be large since there is not a lot of energy in the decay products. There are also α_s corrections [21] and corrections in the Heavy Quark Effective Theory $1/m_b$ expansion [22], the most important being the Fermi motion of the b-quark, which requires the use of another shape function.

Nevertheless, it is possible to obtain the linear combination $M_k^{J/\psi}$ from B decays, with qualitative agreement with the Tevatron extractions [21,22]

$$M_{3,1}^{J/\psi} = 1.5 \times 10^{-2} \text{ GeV}^3, \quad (18)$$

$$M_{3,4}^{J/\psi} = 2.4 \times 10^{-2} \text{ GeV}^3. \quad (19)$$

6. CONCLUSIONS

NRQCD has been used for a number of years to make predictions about quarkonium production. At this time, there are consistent extractions of the NRQCD matrix elements from a number of different experiments. The most analysis has been done on hadronic collisions, in particular at the Tevatron due to the high statistics.

LEP is a very clean place to extract the matrix elements. It is possible obtain smaller theoretical uncertainty compared to the Tevatron extractions. Unfortunately the statistics are not as good. Nevertheless, due to the large theoretical errors in the Tevatron extractions, the LEP extraction is the best extraction to date.

It is also possible to test the NRQCD formalism at LEP by looking at the moments of the spectrum. Without the color octet channels included in the NRQCD formalism, the theory and experiment would not agree for this test. This is one place where NRQCD passes a test. It will be interesting to see whether it will be possible to resolve the polarization problem within the NRQCD factorization formalism.

I would like to thank my collaborators, Glenn Boyd, Peter Cho, and Ira Rothstein, and the organizers of the IV International Conference on Hyperon, Charm and Beauty Hadrons. This work was supported in part by the Department of Energy under grant number DOE-ER-40682-143.

REFERENCES

1. G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. **D51**, (1995) 1125; erratum Phys. Rev. **D55**, (1997) 5853.
2. For recent work on NRQCD, see M. E. Luke, A. V. Manohar and I. Z. Rothstein, Phys. Rev. **D61**, 074025 (2000), and references therein.
3. E. Braaten and S. Fleming, Phys. Rev. Lett. **74** (1995) 3327.
4. P. Cho and M. B. Wise, Phys. Lett. **B346** (1995) 129; M. Beneke and I. Z. Rothstein, Phys. Lett. **B372** (1996) 157; M. Beneke and M. Kramer, Phys. Rev. **D55** (1997) 5269; A. K. Leibovich, Phys. Rev. **D56** (1997) 4412; E. Braaten, B. A. Kniehl and J. Lee, hep-ph/9911436.
5. T. Affolder *et al.* [CDF Collaboration], hep-ex/0004027.
6. M. Krämer, these proceedings.
7. J. L. Domenech and M. A. Sanchis-Lozano, Phys. Lett. **B476** (2000) 65; E. Braaten, S. Fleming and A. K. Leibovich, hep-ph/0008091.
8. P. Cho and A. K. Leibovich, Phys. Rev. **D53** (1996) 150.; Phys. Rev. **D53** (1996) 6203.
9. M. Beneke and M. Kramer, Phys. Rev. **D55** (1997) 5269.
10. B. Cano-Coloma and M. A. Sanchis-Lozano, Nucl. Phys. **B508** (1997) 753.
11. M. A. Sanchis-Lozano, hep-ph/9907497.
12. B. A. Kniehl and G. Kramer, Eur Phys J C6 (1999) 493.
13. J. H. Kühn and H. Schneider, Phys. Rev. **D24** (1981) 2996; Z. Phys. **C11** (1981) 263.
14. V. Barger, K. Cheung and W. Y. Keung, Phys. Rev. **D41** (1990) 1541; E. Braaten, K. Cheung and T. C. Yuan, Phys. Rev. **D48** (1993) 4230.
15. K. Cheung, W. Keung and T. C. Yuan, Phys. Rev. Lett. **76** (1996) 877; P. Cho, Phys. Lett. **B368** (1996) 171.
16. Yu. L. Dokshitzer *et. al.*, “Basics of Perturbative QCD”, Editions Frontiers, Gif-sur-Yvette (1991); A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rep. **100** (1983) 201.
17. C. G. Boyd, A. K. Leibovich and I. Z. Rothstein, Phys. Rev. **D59** (1999) 054016.
18. ALEPH Collaboration, submission to the 1997 EPS-HEP conference, No. 624.
19. I. Z. Rothstein and M. B. Wise, Phys. Lett. **B402** (1997) 346; M. Beneke, I. Z. Rothstein and M. B. Wise, Phys. Lett. **B408** (1997) 373.
20. M. Beneke, G. A. Schuler and S. Wolf, hep-ph/0001062.; S. Wolf, these proceedings, hep-ph/0008180.
21. M. Beneke, F. Maltoni and I. Z. Rothstein, Phys. Rev. **D59** (1999) 054003.
22. J. P. Ma, hep-ph/0006060.